

Hall effect between parallel quantum wires

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We study theoretically the parallel quantum wires of the experiment by Auslaender *et al.* [Science **308**, 88 (2005)] at low electron density. It is shown that a Hall effect as observed in two- or three-dimensional electron systems develops as one of the two wires enters the spin-incoherent regime of small spin bandwidth. This together with magnetic field dependent tunneling exponents clearly identifies spin-incoherence in such experiments and it serves to distinguish it from disorder effects.

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Since its discovery over a century ago the Hall effect has helped to uncover a number of most fundamental physical effects. Among the most famous are the quantization of electrical conductance in the integer quantum Hall effect [1], the fractionalization of electric charge in the fractional quantum Hall effect [2], and anomalous velocities due to Berry phases in ferromagnets [3, 4].

In this Letter we show that, contrary to what one may expect, Hall measurements are also a powerful probe of one-dimensional quantum wires. We predict clear signatures of “spin-incoherent” physics in Hall measurements on tunnel-coupled, parallel quantum wires. The spin-incoherent limit of the interacting one-dimensional electron gas is reached when the temperature T becomes larger than the spin bandwidth J , $kT \gg J$. This regime is a generic property of interacting electrons at low densities, when a Wigner crystal with large inter-electron spacing is formed. As one of the few known regimes of one-dimensional conductors that displays physics qualitatively different from the conventional Luttinger liquid this limit has received much recent theoretical attention [5, 6, 7]. Experimentally, however, it has not been identified conclusively, yet. One of the most promising candidate systems for reaching the low density regime required for observing spin-incoherent physics are the semiconductor quantum wires of the experiment by Auslaender *et al.*, Refs. [8, 9]. The tunneling current in that experiment has shown a loss of momentum resolution at low electron densities. This finding was likely due to a breaking of translational invariance by disorder [9], but it is also the main previously known [10] signature of spin-incoherence in the experimental arrangement of Refs. [8, 9]. An experimental probe that is able to distinguish spin-incoherent physics from the breaking of translational invariance in that experimental setting is thus urgently needed if spin-incoherence is to be observed in such experiments. The Hall measurements proposed here are such a probe [11].

In the experiments of Refs. [8, 9] two parallel one-dimensional wires in a perpendicular magnetic field B are close enough for electrons to tunnel between them, see Fig. 1. A Hall effect in this geometry should induce a

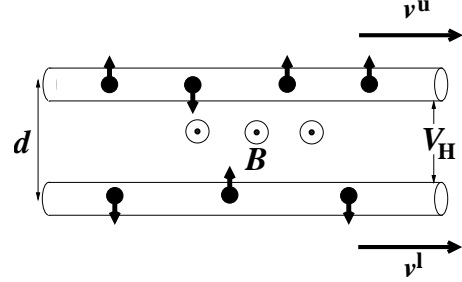


FIG. 1: Two tunnel-coupled, one-dimensional wires at a distance d in a perpendicular magnetic field B . At low densities the conduction electrons form Wigner crystals. The crystals are sliding at velocities v^u and v^l when electrical currents flow. The figure illustrates wires at $J \ll kT$. They have an effectively static spin configuration and an almost conventional Hall voltage V_H appears.

voltage V_H between the two wires in response to a current I flowing through them. For noninteracting electrons in a translationally invariant setup, however, no such voltage is expected. Tunneling then is momentum-resolved and occurs only between a few discrete momentum states. In the generic case that the current I that flows through the wires is not carried by any of the states that participate in the tunneling between them, the tunnel current, and correspondingly V_H , is independent of I . Nevertheless, a transverse voltage can be observed in such experiments if translational invariance is broken or through electron-electron interactions. We show that at $kT \ll J$ the breaking of translational invariance induces a transverse voltage V_H that is generically weak and very unconventional in that it is nonlinear in B . In contrast, in the spin-incoherent regime of $kT \gg J$ a Hall effect as known from higher-dimensional electron systems is found, with a Hall voltage linear in B and I . This clear signature of spin-incoherence, distinguishing it from disorder effects, makes Hall measurements on parallel quantum wires a promising tool in the search for this new and exciting type of one-dimensional physics.

The emergence of traditional Hall physics in spin-incoherent Wigner crystals is due to the nearly classical character of charge transport in this regime. When elec-

trical currents I^μ flow the Wigner crystals are sliding at velocities $v^\mu \propto I$. Here, the index $\mu \in \{u, l\}$ distinguishes the upper from the lower wire in Fig. 1. At $kT \gg J^\mu$ the electrons on the lattice sites of the crystal are distinguishable through the effectively static spins attached to them and therefore behave very similarly to classical,

charged particles. They experience a Lorentz force $\propto I$ that induces an (almost) conventional Hall voltage.

Calculation: To lowest order in the tunnel coupling λ between the wires of a setup as shown in Fig. 1 the tunneling current I_T between them takes the form [12]

$$I_T = e|\lambda|^2 \sum_{\sigma} \int dt dx dx' e^{ieV_T t + iq_B(x-x')} [G_{u\sigma}^>(x, x', t) G_{l\sigma}^<(x', x, -t) - G_{u\sigma}^<(x, x', t) G_{l\sigma}^>(x', x, -t)]. \quad (1)$$

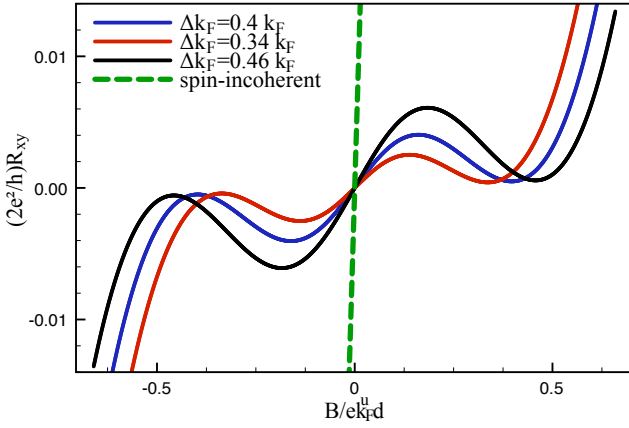


FIG. 2: Transverse resistance R_{xy} of two coupled quantum wires at $I^u = I^l$. At $kT \ll J^\mu$ (solid lines) the dependence on B is nonlinear. In the spin-incoherent case $kT \gg J^\mu$ (broken line), in contrast, R_{xy} is linear in B with a slope greatly exceeding $dR_{xy}/dB|_{B=0}$ at $kT \ll J^\mu$ (solid line: $\Delta k_F l_{br} \gg 1$; broken line: for identical wires).

Here, V_T is the difference between the chemical potentials of the wires (we set $\hbar = 1$). In a magnetic field B the electrons experience a momentum boost $q_B = eBd$ when tunneling between the wires that are a distance d from each other [13]. G_u and G_l are the electron Green functions in the upper and the lower wire respectively. They depend on the currents I^μ that flow through the wires.

Broken translational invariance: We first consider the case that translational invariance is broken, but $kT \ll J^\mu$, such that the wires have not entered the spin-incoherent regime. At sufficiently low energies such wires are described by Luttinger liquids [14] with Fermi wavevectors k_F^μ , Fermi velocities v_F^μ and interaction parameters g_c^μ and g_s^μ of their charge and spin modes respectively [15]. We assume that translational invariance is broken over a length l_{br} that is shorter than the electron wavepackets such that l_{br} shows in observables, $eV, kT \ll v_F^\mu/l_{br}$, where $V = \max\{V_T, I^u/e, I^l/e\}$.

In the experiments of Refs. [8, 9] momentum conservation is typically lifted through the finite length of the tunneling region, disorder, or a leakage of electrons into the surrounding two-dimensional electron gas with mean free path l_{1D-2D} . We first assume that the latter is the dominant mechanism, such that $l_{br} = l_{1D-2D}$. At $eV \ll kT$, $v_F^\mu|q_B \pm k_F^u \pm k_F^l|$ we then find

$$I_T \propto T^\alpha \sum_{\sigma^u, \sigma^l = \pm 1} f(\sigma^u k_F^u + \sigma^l k_F^l - q_B) \times \left(\pi \frac{\sigma^u I^u + \sigma^l I^l}{2e^2} - V_T \right) \quad (2)$$

with $\alpha = -1 + \sum_{\nu \in \{c, s\}} (g_\nu^u + g_\nu^{u-1} + g_\nu^l + g_\nu^{l-1})/4$ and $f(k) = l_{br}/(1 + k^2 l_{br}^2)$. The transverse voltage V_H is found as the counter voltage $V_H = -V_T$ needed to cancel the tunneling current, $I_T = 0$. When $I^u = I^l$, mimicking the higher-dimensional case, we find a transverse resistance $R_{xy} = V_H/I$, where $I = I^u + I^l$, of

$$R_{xy} = \frac{\pi q_B}{e^2 (2k_F^u)^3} \frac{\prod_{\sigma=\pm 1} [(\Delta k_F - \sigma q_B)^2 + l_{br}^{-2}]}{\Delta k_F^2 + q_B^2 + l_{br}^{-2}} \quad (3)$$

at $|\Delta k_F|, q_B, l_{br}^{-1} \ll k_F^u$ ($\Delta k_F = k_F^u - k_F^l$). We make two observations: i) R_{xy} is nonlinear in B on the scale $\Delta B \sim \max\{|\Delta k_F|/ed, (ed l_{br})^{-1}\}$, as illustrated in Fig. 2; ii) the ‘differential Hall coefficient’ $dR_{xy}/dB|_{B=0} = R_H^{(0)} \times [\Delta k_F^2 + 1/l_{br}^2]/(2k_F^u)^2$ is suppressed below the Hall coefficient $R_H^{(0)} = -1/en_{2D}$ that one would expect in a two-dimensional electron gas. Here, $n_{2D} = (n^u + n^l)/d$ is an effective two-dimensional electron density between the two wires with one-dimensional densities $n^\mu = 2k_F^\mu/\pi$. Also the Hall response $R_{xy}^{(-)}$ to a difference $I^{(-)} = I^u - I^l$ between the currents through the wires, $R_{xy}^{(-)} = -\pi q_B \Delta k_F / e^2 (\Delta k_F^2 + q_B^2 + l_{br}^{-2})$ (again at $|\Delta k_F|, q_B, l_{br}^{-1} \ll k_F^u$), where $V_H = R_{xy} I + R_{xy}^{(-)} I^{(-)}$, is nonlinear in B on the scale ΔB . The differential Hall response to a difference in currents $dR_{xy}^{(-)}/dB|_{B=0} = [-8\Delta k_F k_F^3 / (\Delta k_F^2 + l_{br}^{-2})^2] \times dR_{xy}/dB|_{B=0}$, however, is strongly enhanced. Other mechanisms for the lifting of

momentum conservation are described by Eq. (2) with a (possibly) different f . Both of our main conclusions hold for any kind of translational invariance breaking and also in the regime $v_s/l_{br} \gg eV_T, I^u/e, I^l/e \gg kT$.

One spin-incoherent wire: We next discuss the situation that the upper wire has a low electron density, $k_F^u < k_F^l$, and exhibits spin-incoherent physics, $kT \gg J^u$, while the lower wire is still described by a conventional Luttinger liquid, $kT \ll J^l$. This is motivated by the experiment of Ref. [9], where the observed loss of momentum conservation was attributed to only one of the two wires. We model the spin-incoherent upper wire following Refs. [16, 17]. Its Green function after the spin trace takes the form [16, 17]

$$G_{u\sigma}^>(x, x', \tau) = -i \int \frac{d\xi}{2\pi} dk p_\sigma^{|k|} e^{i\xi k} \quad (4)$$

$$\times \langle e^{-i\xi N_x(\tau)} c^\dagger(x, \tau) c(x', 0) e^{i\xi N_{x'}(0)} \rangle,$$

and similarly for $G_{u\sigma}^<$. Here, c are spinless fermions that form a Luttinger liquid with interaction parameter $g^u < 1$ inside the wire and N_x is the number of fermions c to the right of point x . We describe a current-carrying spin-incoherent wire of finite length L contacted by non-interacting leads following Ref. [18] and evaluate Eq. (4) by bosonization of the fermions c . Via the x -dependence

of N_x the integrations in Eq. (4) generate a space dependence of fermionic amplitudes on the scale $(k_F^u)^{-1}$. Since with our bosonization approach we access only the long wavelength limit we assume that a magnetic field is applied in the plane of the wires that favors one of the spin states, $1 - p_\uparrow \ll 1$. The space dependence in Eq. (4) is then on the length scale $(k_F^u \ln p_\uparrow)^{-1} \gg (k_F^u)^{-1}$. We expect, however, all results to remain qualitatively valid also at $p_\uparrow \approx p_\downarrow$. We only evaluate $G_{u\uparrow}$ here since the minority spin tunnel current is expected to be negligible.

In the following we address the regime of moderately low voltages, $kT, v_F^u/L \ll eV \ll \ln p_\uparrow/\delta$ with $\delta \sim 1/v_F^u k_F^u$. In this regime we obtain

$$G_{u\uparrow}^>(x, x', \tau) = \frac{n^u e^{i\pi I^u(x-x')/ev_F^u}}{\sqrt{2\pi g \ln[(i\tau + \delta)/\delta]}} \left(\frac{\delta}{i\tau + \delta} \right)^{1/2g^u} \quad (5)$$

$$\times \int dk p_\uparrow^{|k|} \cos \pi k e^{-\pi^2 [k - I^u \tau / e - (x-x')n^u]^2 / 2g \ln[(i\tau + \delta)/\delta]}$$

[at $1/kT \gg \tau \sim 1/eV \gg \delta/\ln p_\uparrow$], where now $n^u = k_F^u/\pi$. As a consequence of spin-incoherence, G_u decays quickly as a function of $x - x'$. Assuming that this is the dominant mechanism for the lifting of momentum conservation, $\max\{1/k_F^u \ln p_\uparrow, \sqrt{-g \ln eV_T \delta / k_F^u}\} \ll l_{br}$, we then find from Eqs. (5) and (1) that

$$I_T \sim \sum_{\sigma^u, \sigma^l = \pm 1} \frac{\ln p_\uparrow}{\ln^2 p_\uparrow + \pi^2 [\sigma^u + (q_B/k_F^u) + (\sigma^l k_F^l/k_F^u)]^2} \left[-V_T + \frac{\pi I^u}{e^2} \left(\frac{q_B}{k_F^u} + \frac{\sigma^l k_F^l}{k_F^u} \right) - \sigma^l \frac{\pi I^l}{2e^2} \right]^{\alpha_{\sigma^l}} \quad (6)$$

with the scaling exponents

$$\alpha_\sigma = \frac{1}{2g^u} + \frac{g^u}{2} \left(\frac{q_B}{k_F^u} + \frac{\sigma k_F^l}{k_F^u} \right)^2 - 1 + \sum_{\nu \in \{c, s\}} \frac{1}{4g_\nu^l} + \frac{g_\nu^l}{4}. \quad (7)$$

In our limit $1 - p_\uparrow \ll 1$, the first factor in Eq. (6) consistently suppresses large momentum transfers $q_B + \sigma^u k_F^u + \sigma^l k_F^l$, where our bosonization calculation is unreliable. For simplicity we now assume that the denominator $\ln^2 p_\uparrow + \pi^2 [\bar{\sigma}^u + (q_B/k_F^u) + (\bar{\sigma}^l k_F^l/k_F^u)]^2$ of the summand in Eq. (6) with $\bar{\sigma}^u, \bar{\sigma}^l = \pm 1$ is much smaller than the denominators in all other summands such that all but this one summand may be neglected.

We first note that, in contrast with the conventional Luttinger liquid, the tunneling current as a function of the applied voltages obeys a power law with an exponent $\alpha_{\bar{\sigma}}$ that depends on the magnetic field B . The B -dependence of $\alpha_{\bar{\sigma}}$ is due to a Fermi-edge singularity [19, 20] with scattering phase shift $\delta\varphi = (\bar{\sigma}^u k_F^u + \bar{\sigma}^l k_F^l + q_B)/n^u$. To understand the origin of this phase shift we analyze the tunneling rate, given by amplitudes for the

addition of an electron to the wire multiplied by complex conjugated amplitudes, describing the removal of an electron. As a consequence of spin-incoherence, these pairs of amplitudes are constrained to add and remove a spin at the same site of the spin configuration of the Wigner crystal (otherwise the spin expectation values are suppressed by powers of p_\uparrow). Suppose that an electron in the Wigner crystal crosses the point of tunneling during the time between the addition and the removal of a tunneling electron. This shifts the spin background by one lattice site. The above constraint can thus only be satisfied if the locations for the addition and the removal of the tunneling electron in space differ by the inter-electron distance $\Delta x = 1/n^u$. The phase $(\bar{\sigma}^u k_F^u + \bar{\sigma}^l k_F^l + q_B)\Delta x$ that the tunneling electron picks up as a result translates into the effective phase shift $\delta\varphi$ for the electron of the Wigner crystal that crossed the point of tunneling.

When a current I^u flows through the upper wire (at $I^l = 0$), the upper crystal slides at velocity $v^u = I^u/en^u$. So does the point of tunneling, which makes the phase shift $\delta\varphi$ time-dependent and thus induces a (Hall) voltage

between the wires. As before we find from Eq. (6) that

$$V_H = (BR_H + R'_{xy}) I^u. \quad (8)$$

The first term in Eq. (8) remarkably describes a conventional Hall effect as known from higher dimensions with $R_H = R_H^{(0)}$ at $n_{2D} = n^u/d$ (n^l does not enter n_{2D} since the lower wire does not participate in the Hall effect). The second contribution to V_H , proportional to $R'_{xy} = -\bar{\sigma}^l k_F^l / e^2 n^u$, resembles the anomalous Hall resistance in ferromagnets and does not vanish at $B = 0$. Its origin is best understood in the reference frame comoving with the sliding Wigner crystal in the upper wire. In that frame the energies of the electrons at the two Fermi points $\sigma^l = \pm 1$ of the lower wire are shifted relative to those in the rest frame by $v^u \sigma^l k_F^l$ through a Galilean boost. The resulting shift in chemical potential results in the extra voltage described by R'_{xy} . Note that Eq. (8) is invalid in zero magnetic field since our above assumption that one summand in Eq. (6) dominates cannot be satisfied. In zero magnetic field one finds $R'_{xy} = 0$, so no anomalous Hall effect as in ferromagnets can be observed in this system. Current flow in the lower wire does not modify the Hall coefficient, but only changes R'_{xy} .

Two spin-incoherent wires: We now analyze the situation that both wires are spin-incoherent, $kT \gg J^\mu$. At low voltages $|\ln(eV\delta)| \gg \pi^2/g(\ln p_\uparrow)^2$, $kT \ll eV$, we have

$$I_T \sim \sum_{\sigma^u, \sigma^l = \pm 1} \frac{\ln p_\uparrow}{\ln^2 p_\uparrow + \pi^2 [\sigma^u + \bar{g} q_B / g^u k_F^l]^2} \times \{u \leftrightarrow l\} \times \left[-V_T + q_B \bar{g} \left(\frac{\pi I^u}{e^2 g^u k_F^l} + \frac{\pi I^l}{e^2 g^l k_F^u} \right) \right]^\alpha \quad (9)$$

with $\bar{g} = g^u g^l n^u n^l / [g^u (n^l)^2 + g^l (n^u)^2]$ and $\alpha = 1/2g^u + 1/2g^l + \bar{g} q_B^2 / 2k_F^u k_F^l - 1$. We find

$$V_H = B \left[R_H I + R_H^{(-)} I^{(-)} \right] \quad (10)$$

[21]. Unlike Eq. (8), that was derived under a B -dependent condition that allowed to neglect terms in Eq. (6), Eq. (10) predicts a V_H linear in B in the entire range of validity of our bosonization approach (set by the scale $\min\{k_F^u, k_F^l\}$). This contrasts clearly with the conventional Luttinger liquid regime, where V_H becomes nonlinear on the scale Δk_F , as shown in Fig. 2. The Hall coefficient $R_H = -(\bar{g}d/2e)(1/g^u n^l + 1/g^l n^u)$ is again of the order of the classically expected one and thus strongly enhanced compared to the conventional Luttinger liquid (see Fig. 2). The magnitude of the Hall response to the difference between the currents through the two wires $R_H^{(-)} = -(\bar{g}d/2e)(1/g^u n^l - 1/g^l n^u)$ is now smaller than R_H , while it had been found to be strongly enhanced in the absence of spin-incoherence. Counter-intuitively, the Hall response to currents in the wire with the lower

electron density (found as $R_H \pm R_H^{(-)}$ with the positive sign if the upper wire has smaller density than the lower wire) is smaller than the one in the wire with higher density - although the lower density crystal slides faster and experiences a stronger Lorentz force at $I^u = I^l$. The conventional relation $V_H = R_H^{(0)} I$ holds only if both crystals slide at the same velocity $v^u = I^u / en^u = I^l / en^l = v^l$. Also these features are readily understood by analyzing the rate of tunneling between the wires. The addition and the removal of an electron in each pair of amplitudes that contributes to it typically occur within a time $t_T \sim 1/eV_T$. Spin-incoherence again constrains the two amplitudes for adding and removing a spin to act at the same site of the spin configuration of each wire. If $v^u \neq v^l$, however, the spin configurations of the two wires are diverging in space at the average speed $v^u - v^l$. After the time t_T they can be aligned only if the two crystals are compressed by amounts Δx^u and Δx^l with $\Delta x^u - \Delta x^l = -(v^u - v^l)t_T$. This costs an elastic energy $\epsilon_{\text{elastic}} \propto (n^u \Delta x^u)^2 / g^u + (n^l \Delta x^l)^2 / g^l$. Maximizing the probability $\exp(-S)$ of the corresponding deformation, where $S \propto \epsilon_{\text{elastic}}$, under the constraint $\Delta x^u - \Delta x^l = -(v^u - v^l)t_T$ we find $\Delta x^u = -t_T(v^u - v^l)\bar{g}n^l/n^u g^l$. This distortion of the crystals results in a modified effective velocity of an electron during the tunneling process of $v_{\text{eff}} = -(R_H I + R_H^{(-)} I^{(-)})/d$. The corresponding Lorentz force implies Eq. (10). Now the reason for the suppression of the Hall coefficient of the low-density wire noted above is evident: because the electron configuration in the low-density wire is deformed more easily v_{eff} (and thus V_H) is predominantly determined by the wire with the higher density and depends only weakly on the current through the low-density wire.

Conclusions: We have studied tunneling between parallel quantum wires at low electron density. An almost conventional Hall effect has been shown to emerge as the wires enter the spin-incoherent regime of small spin bandwidth. The Hall coefficient is of the order of the one classically expected at a given electron density and the Hall voltage only weakly depends on the difference of the currents through the two wires. In contrast, two wires in the absence of spin-incoherence with weak translational symmetry breaking, $\Delta k_F l_{\text{br}} \gtrsim 1$, have a Hall coefficient that is suppressed under its classical value by a factor of $(\Delta k_F / k_F)^2$, where Δk_F is the difference between the Fermi wavevectors of the two wires with average wavevector k_F , while the Hall response to a difference between the currents that flow through such wires is anomalously enhanced by a factor $(k_F / \Delta k_F)^3$ compared to the response to the average current. Moreover, wires in the conventional regime exhibit a nonlinear magnetic field dependence on the scale set by Δk_F (again for $\Delta k_F l_{\text{br}} \gtrsim 1$). In contrast, spin-incoherent conductors are predicted to produce a transverse voltage that is linear in the magnetic field up to a scale

of the order of the Fermi wavevectors themselves. This together with magnetic field dependent tunneling exponents clearly identifies spin-incoherent physics in experiments like those of Refs. [8, 9]. In particular, it distinguishes spin-incoherence from the effects of disorder. Such measurements are thus a very promising avenue in the search for this novel regime of interacting quantum wires.

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